Eigenvalues first?

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Special thanks to Ben Woodruff at BYU-Idaho

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emphasize fundamental concepts throughout the entire course

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• concrete computation \rightarrow intuition \rightarrow formal generalization

Course

Goals

Course Materials

- 1. Computations and brief explanation
- 2. Application: motivation and practice

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3. Theory and generalization

1. Computation and brief explanation

- matrix/vector operations
- RREF and solving systems
- determinants
- inverses
- linear dependence
- spans
- bases and coordinates
- rank
- eigenvalues/eigenvectors

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Purpose: Motivation and practice

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Concept	matrix mult.		solving	evals	pretty
Арр	cols	rows	systems	evecs	pictures
Multivariable Optimization				х	
Vector Fields		х		х	
Markov Models	Х			Х	
Kirchoffs' Laws		Х	Х		
Interpolating Polynomials		Х	Х		
Least Squares	х	Х	Х		

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Practice

- Cramer's rule: determinants, inverses
- Least Squares: dot product, angle, projections, transpose, coordinates, bases
- Finding standard bases: bases, coordinates, column space, row space, RREF

- New concepts
 - Least Squares: column, row, and null spaces

Vector Spaces and Matrix Theorems

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- Inner Products (nice bases)
- Linear Transformations
- Changing Bases

Course

Goals

Course Materials



emphasize fundamental concepts throughout the entire course

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• concrete computation \rightarrow intuition \rightarrow formal generalization

Emphasis: Eigenvalues and Eigenvectors

- 1. Computation: basic concepts and computation
- 2. Applications: 3 major applications (optimization, vector fields, Markov models)
- 3. Patterns and Vector Spaces: formal footing and relationships
- 4. Inner Products: inner products on \mathbb{R}^n
- 5. Linear Transformations: Geometry, connection to null space and determinants

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6. Changing Bases: Diagonalization

Mindshare: entire course

- 1. Computation: basic concepts and computation
- 2. Applications: application and student project
- 3. Patterns and Vector Spaces: formal/generic context
- 4. Inner Products: orthogonal basis
- 5. Linear Transformations: Finding matrices for linear transformations

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6. Changing Bases: Fundamental concept

Mindshare: entire course

 $\mbox{Goal: concrete computation} \rightarrow \mbox{intuition} \rightarrow \mbox{formal generalization}$

- 1. Introduce vector subspaces as spans of vectors in \mathbb{R}^n
- 2. Introduce row, column, and null space as vector subspaces
- 3. Patterns and Vector Spaces
 - ► Review subspaces as spans of vectors, cover subspace theorem

- ► Generalize "vector": polynomial, matrix vector spaces
- Generalize "vector addition" and "scalar multiplication": general vector spaces
- 4. Inner Products: Use function vector spaces

Course

Goals

Course Materials





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- Many homework problems and chapter projects written
- Not enough, though

Schaum's Beginning Linear Algebra

- \$12.63 on Amazon
- Brief explanations and examples
- ▶ 652 fully-solved homework problems



groups.google.com/group/math-problembank

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Sage: Free Open Source Math Software



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Mission Statement

Creating a viable free open source alternative to Magma, Maple, Mathematica and Matlab.

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Introductory worksheets and classroom aids

2009, 2010 Ben Woodruff, BYU-Idaho: Wrote initial textbook Fall 2010 Jason Grout, Drake University: Revised, reordered, corrected, augmented text

— We are Here —

Winter 2011 Jason Grout, Continue revising

- Revise order of some topics
- Continue to add new content and exercises

- Sage worksheets
- Listen to YOUR suggestions

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Thank You!

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Book: artsci.drake.edu/grout/doku.php/books

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